J. Phys. A: Math. Gen. **39** (2006) 4401–4405 doi:10.1088/0305-4470/39/17/014

Optical HF electrical permeability, refractivity and reflectivity of dense non-ideal plasmas

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Received 12 September 2005, in final form 1 December 2005
Published 7 April 2006
Online at stacks.iop.org/JPhysA/39/4401

Abstract
On the basis of numerically calculated values for the dense plasma dynamic conductivity in an external HF electric field, we determine the electrical permeability and the coefficients of refractivity and reflectivity of dense non-ideal plasmas. We consider electronic number density and temperature in the ranges of $10^{21} \leqslant N_e \leqslant 10^{23}$ cm^{-3} and 20 kK $\leqslant T \leqslant$ 1 MK, respectively. The parametrized form of representation of the results is suitable for experimental verification and further usage. The examined range of frequencies covers the IR, visible and near UV regions.

PACS numbers: 51.70.+f, 52.27.Gr

Recently, in [1, 2], we obtained numerical data for the HF electrical permeability $\varepsilon(\omega)$ and the coefficients of refractivity $n(\omega)$ and reflectivity $R(\omega)$ of slightly and moderately non-ideal plasmas, with ω being the frequency of the external electric field. This work was based on the extended Drude–Lorentz model for the plasma dynamic conductivity

$$\sigma(\omega) = \frac{4e^2}{3m}\int_0^\infty \frac{\tau(E)}{1 - \mathrm{i}\omega\tau(E)} \cdot \left[-\frac{\mathrm{d}w(E)}{\mathrm{d}E}\right]\rho(E)E\,\mathrm{d}E, \tag{1}$$

where $\rho(E)$ is the density of electronic states in the energy space, $w(E)$ is a Fermi–Dirac distribution function and $\tau(E)$ is the static electronic relaxation time. The latter is the basic parameter of our theory, which is evaluated within a self-consistent field quantum-statistical approach going beyond the random-phase approximation; for more details see [3–6] and also [7].

All these characteristics were calculated in [1, 2] in the electronic number density range $N_e \leqslant 10^{21}$ cm^{-3}. However, in our work [7] the values of the complex $\sigma(\omega)$ were determined in the region $10^{21} \leqslant N_e \leqslant 10^{23}$ cm^{-3}. This enables us to obtain the data for $\varepsilon(\omega)$, $n(\omega)$ and $R(\omega)$ in the same region of N_e. Here we present the results of our computations carried out in this extended electronic density interval and for the plasma temperature varying in the range

$20\,000 \leqslant T \leqslant 1000\,000$ K. In this area the non-ideality parameter $\Gamma = \beta e^2/a \in [0.03\text{–}6.30]$. Here, as usual, $\beta = (kT)^{-1}$ and $a = (4\pi N_e/3)^{1/3}$ is the electronic Wigner–Seitz radius. Similarly to the previous papers, we consider completely ionized hydrogen or hydrogen-like plasmas.

It was suggested in [1, 2] that the computed data for the real, $\sigma_{\mathrm{Re}}(\omega)$, and imaginary, $\sigma_{\mathrm{Im}}(\omega)$, parts of the complex dynamic conductivity (1) could be parametrized (for $\omega \geqslant 0$) in the form

$$\sigma_{Re}(\omega) = \frac{\sigma_0}{1 + \left(\frac{\omega}{\omega_p}\right)^2 f_{0p}^2 k_1^2}, \qquad \sigma_{Im}(\omega) = \frac{\sigma_0\left(\frac{\omega}{\omega_p}\right) f_{0p} k_2}{1 + \left(\frac{\omega}{\omega_p}\right)^2 f_{0p}^2 k_1^2}. \tag{2}$$

Here $\sigma_0 = \sigma_{Re}(0)$ is the static conductivity, $f_{0p} = \omega_p \tau_0^* = 4\pi\sigma_0/\omega_p$, τ_0^* is the effective relaxation time, $\omega_p = \sqrt{4\pi N_e e^2/m}$ is the plasma frequency and k_1 and k_2 are parameters which describe the deviation from the classical Drude–Lorentz model: if both k_1 and k_2 tend to 1, we return to

$$\sigma_D(\omega) = \frac{\sigma_0}{1 - \mathrm{i}\omega\tau_0^*}. \tag{3}$$

The corresponding values of σ_0 and f_{0p} are presented in [7], and the values of the factors k_1 and k_2 are determined from equations (1) and (2); as shown in [1, 2], they can be cast in the following approximate fitting form:

$$k_j = k_j\left(\frac{\omega}{\omega_p}\right) = k_{j0} - \frac{a_j^2 b_j\left(\frac{\omega}{\omega_p}\right)}{1 + a_j b_j\left(\frac{\omega}{\omega_p}\right)}, \qquad j = 1, 2, \tag{4}$$

where the adjustment parameters k_{10}, a_1, b_1 and k_{20}, a_2, b_2 are determined numerically from the condition of best agreement between the 'exact' and fitted values of the factors k_1 and k_2, i.e., between the conductivity values found from (1) and (2).

Thus the plasma dielectric permeability is

$$\epsilon(\omega) = 1 + \mathrm{i}\frac{4\pi}{\omega}\sigma(\omega) = \epsilon_{Re}(\omega) + \mathrm{i}\epsilon_{Im}(\omega), \tag{5}$$

and the coefficients of refraction, $n(\omega)$, and reflection, $R(\omega)$, are determined as

$$n(\omega) = \sqrt{\epsilon(\omega)} = n_{Re}(\omega) + \mathrm{i}n_{Im}(\omega), \qquad R(\omega) = \left|\frac{n(\omega) - 1}{n(\omega) + 1}\right|^2. \tag{6}$$

The other parameter of interest, which was not treated in [1], is the penetration depth of electromagnetic radiation into plasma, $\Delta(\omega)$. This quantity is just the skin-layer width determined as the inverse imaginary part of the electromagnetic field wave number [8]:

$$\Delta(\omega) = \frac{1}{k_{Im}} = \frac{c}{\omega}\frac{1}{n_{Im}}, \tag{7}$$

with c being the vacuum light speed.

It stems from (2) that

$$\epsilon_{Re}(\omega) = 1 - \frac{4\pi}{\omega_p}\frac{\sigma_0 f_{0p} k_2}{1 + \left(\frac{\omega}{\omega_p}\right)^2 f_{0p}^2 k_1^2}, \qquad \epsilon_{Im}(\omega) = \frac{4\pi}{\omega}\frac{\sigma_0}{1 + \left(\frac{\omega}{\omega_p}\right)^2 f_{0p}^2 k_1^2}. \tag{8}$$

Similar parametrized forms for n_{Re}, n_{Im}, R and Δ can be easily obtained as well.

Though the fitting parameters k_1 and k_2 represent only the measure of deviation of the actual dynamic conductivity from the Drude–Lorentz model (3), the latter expression for the complex dielectric function of non-ideal plasmas is of practical importance.

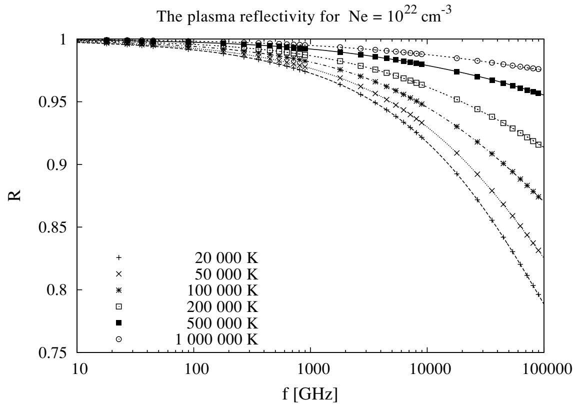

Figure 1. The plasma reflectivity for $N_e = 10^{22}$ cm^{-3} and 2×10^4 K $\leqslant T \leqslant 10^6$ K. The values calculated using the data on the dynamic conductivity (1) are presented by the corresponding symbols; the lines are obtained with the parametrized expressions.

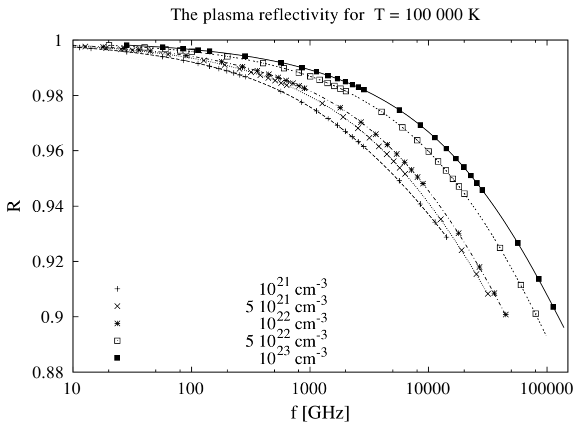

Figure 2. The plasma reflectivity for $T = 10^5$ K and 10^{21} cm^{-3} $\leqslant N_e \leqslant 10^{23}$ cm^{-3}. The values calculated using the data on the dynamic conductivity (1) are presented by the corresponding symbols; the lines are obtained with the parametrized expressions.

In the present work we have employed the results of our calculations of the dynamic conductivity parametrized by means of the parameters k_1 and k_2 performed in [7]; it allows us to analyse the behaviour of the plasma reflectivity coefficient and the penetration length in the chosen domains of N_e, T and ω.

Note that the adjustment parameters $k_{10}, k_{20}, a_1,\ b_1, a_2$ and b_2 were obtained from a standard fitting procedure in the whole area of variation of N_e, T and ω. The important property of these parameters is that they do not depend on frequency ω, and are slowly varying functions of N_e and T. In particular, for $N_e = 10^{22}$ cm^{-3} and when T increases from 10 000 K to 100 000 K, the parameters k_{10} and k_{20} increase monotonically from 1.117 to 1.861 and 1.077 to 1.564, respectively. Simultaneously, the parameters a_1 and a_2 grow monotonically from 0.362 to 1.217 and from 0.379 to 1.105, respectively, while b_1 and b_2 do so from 1.716 to 5.249 and 1.435 to 4.842. The HF characteristics, R and Δ, were thus re-calculated using

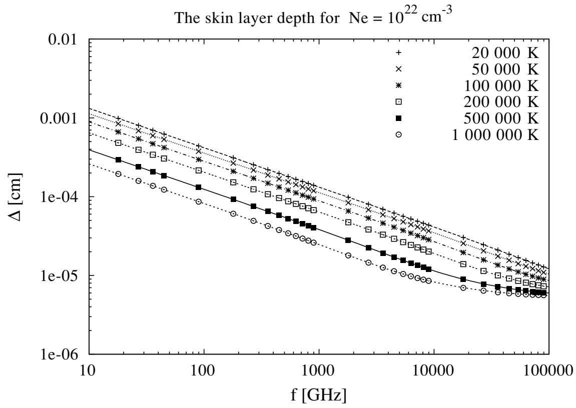

Figure 3. The skin layer depth for $N_e = 10^{22}$ cm^{-3} and 2×10^4 K $\leqslant T \leqslant 10^6$ K. The values calculated using the data on the dynamic conductivity (1) are presented by the corresponding symbols; the lines are obtained with the parametrized expressions.

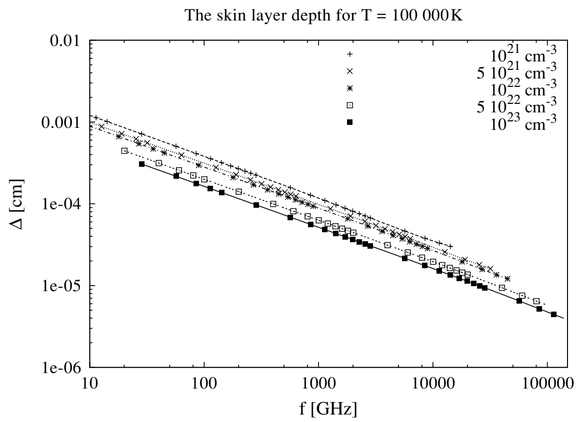

Figure 4. The skin layer depth for $T = 10^5$ K and 10^{21} cm^{-3} $\leqslant N_e \leqslant 10^{23}$ cm^{-3}. The values calculated using the data on the dynamic conductivity (1) are presented by the corresponding symbols; the lines are obtained with the parametrized expressions.

the parametrized formulation for the computed values of σ_0 and the analytic approximations (4) for k_1 and k_2.

The behaviour of the fitted values for optically measurable values of the reflectivity coefficient $R(\omega)$ and the skin layer depth $\Delta(\omega)$ as functions of N_e and T is compared in figures 1–4 to their 'exact' values computed from our model expressions (1) and (5)–(7). In these figures the values calculated using the data on the dynamic conductivity, obtained with the help of the basic equation (1), are presented by the corresponding symbols, and the values obtained with the parametrized expressions are displayed by the lines.

Let us presume that the (monochromatic and orthogonally incident) electromagnetic radiation is reflected from a flat surface of a plasma layer. Then we observe that since for the long-wavelength $\left(\lambda = \frac{c}{f} = \frac{2\pi c}{\omega} \to \infty\right)$ radiation the imaginary part of the dielectric function ((5) or (8)) diverges, i.e., $|\epsilon(\omega \to 0)| \gg \epsilon_{Re}(\omega)$, the reflectivity coefficient $R(\omega \to 0) \to 1$:

the plasma layer acts as a mirror, while there is no significant skin-effect and the penetration length $\Delta(\omega \to 0) \to 0$.

The results we present might be compared to the corresponding experimental data, when available.

Acknowledgments

This work is a part of the project 1466 'Radiation and transport properties of non-ideal laboratory and ionospheric plasmas' of the Ministry of Science, Technology and Development of Serbia.

References

[1] Mihajlov A A, Djurić Z, Adamyan V M and Sakan N M 2001 *J. Phys. D: Appl. Phys.* **34** 3139–44
[2] Adamyan V M, Djurić Z, Mihajlov A A, Sakan N M and Tkachenko I M 2004 *J. Phys. D: Appl. Phys.* **37** 1896–903
[3] Djurić Z, Mihajlov A A, Nastasyuk V A, Popović M and Tkachenko I M 1991 *Phys. Lett.* A **155** 415
[4] Adamyan V M, Djurić Z, Ermolaev A M, Mihajlov A A and Tkachenko I M 1994 *J. Phys. D: Appl. Phys.* **27** 111
[5] Adamyan V M, Djurić Z, Ermolaev A M, Mihajlov A A and Tkachenko I M 1994 *J. Phys. D: Appl. Phys.* **27** 927
[6] Tkachenko I M and Fernández de Córdoba P 1998 *Phys. Rev.* E **57** 2222
[7] Adamyan V M, Mihajlov A A, Sakan N M, Šulić D, Srećković V A and Tkachenko I M 2006 *J. Phys. A: Math. Gen.* **39** 4693–7
[8] Alexandrov A F, Bogdankevich L S and Rukhadze A A 1984 *Principles of Plasma Electrodynamics* (Berlin: Springer)