The Modeling of the Continuous Emission Spectrum of a Dense Non-ideal Plasma in Optical Region

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Abstract. This paper presents a study of the usability of the Coulomb cut-off potential for the calculation of bound-free and free-free cross sections. It covers the condition in dense plasma where there is no good theory that covers the optical properties of dense plasma. The presented quantum mechanical model has given a good agreement with the experimental data. The ideas for further development of the presented calculations are shown.

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THE ION INSIDE PLASMA

The process of radiation and absorption of a light inside plasma is very complex. Since it is not so simple, and sometimes it is impossible, to describe such a complex system behavior, the presented model is based upon the description of a behavior of an ion inside plasma. If an atom absorbs a photon and ionizes, the produced electron is under the influence of the potential of that ion (two-particle interaction) and plasma (many body interaction). Similarly, in a case of free-free absorption, all of the processes of absorption of a photon took place inside plasma potential. The potential of an ion in which vicinity such a process occurs is the one that has most significant influence on the process itself. In both cases the behavior of plasma is modeled with the help of quantum mechanical model, where the many body problem is described as an influence of such a complex system on a potential of an ion. Such influence also reflects on a single atom since the single bound electron is also inside of exactly the same ion potential. Instead of solving many-body problems the procedure is shifted towards two-body system what is much simpler. The influence of plasma on such system is described as a change of the Coulomb potential. The advantage of such an approach is that a dipole approximation is valid in such description, but such an approach forces us to describe a plasma where in the vicinity of an ion is only one electron in average. Simply speaking the description of plasma behavior is accomplished by much simpler two-body model, in which the influence of many bodies on an atom or ion is described as a modification of the Coulomb potential.

Coulomb cut-off potential

If the influence of plasma on the ion inside of it is considered, in general, at the large distance from the ion center the micro-field inside of plasma could be averaged. Also, with the increasing of the distance the potential of the single ion considered is diminishing due to the effects of screening inside of plasma. The simplest model that describes such an influence on plasma is a Cut-off Coulomb potential, given by

$$U(r) = \begin{cases} -\dfrac{e^2}{r} + \dfrac{e^2}{r_c} & : & 0 < r \le r_c \\ 0 & : & r_c < r \end{cases} . \tag{1}$$

This model introduces the cut-off energy $E_{co} = e^2/r_c$ that is an average plasma influence on the ion potential. Also the potential of an ion is not effective after cut-off radius r_c. The missing part in this model is an influence of micro-field. Basically this model is good in a vicinity of ion where the Coulomb potential is too high, so the shifting for some finite value does not influence the solution. Also the quantum mechanical model is also good for the radii that are considerably greater than the cut-off radius r_c. In that case the effect of many micro-fields on a potential are summarized, and a constant potential shift is a good model.

The solutions for the Coulomb cut-off potential

In spite of the obviously missing part in this description, it has some advantages over some more complex models. All of the solutions of the Schroedinger equation are analytical functions. Instead of solving the radial part of a Schroedinger equation for this system

$$\left\{ \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + \frac{2m}{\hbar^2}[E - U(r)] - \frac{l(l+1)}{r^2} \right\} R = 0, \tag{2}$$

we first introduce P(r) given by

$$R(r) = \frac{P(r)}{r}, \tag{3}$$

This transformation give a more easily solvable differential equation

$$\frac{d^2 P(r)}{dr^2} + \left[\frac{2m}{\hbar^2}(E - U(r)) - \frac{l(l+1)}{r^2} \right] P(r) = 0. \tag{4}$$

If the potential (1) is introduced into Eq. (4) the solutions for this equation could be divided in three ranges concerning the energy of the states

a) $E < 0$
b) $0 < E < e^2/r_c$
c) $e^2/r_c < E$

Within the first range the solutions are bound states, with discrete energetical spectrum of the solutions. The second and the third range present the solutions for the free states with continuous energy of free states.

In the first area, the solution is a combination of a Whittaker special function and modified Bessel one

$$P(r) = \begin{cases} C_N M_{\kappa,\mu}\left(\dfrac{2r}{a_0\nu^2}\right) & : \quad r < r_c \\ \sqrt{r}\,C_N C_{E,l} k_{l+1/2}\left(r\sqrt{-2mE/\hbar^2}\right) & : \quad r \geq r_c \end{cases} \tag{5}$$

Here $\mu = l + 1/2$, $\nu = \sqrt{-\dfrac{e^2}{a_0}\dfrac{1}{2E_c}}$, $a_0 = \hbar^2/me^2$ and $E_c = E + e^2/r_c < 0$.

The solution for the second range is given by

$$P(r) = \begin{cases} C_0 M_{\kappa,\mu}\left(\dfrac{2r}{a_0\nu^2}\right) & : \quad r < r_c \\ C_0 r\left[C_1 \dfrac{\sqrt{2mE}}{\hbar} j_l\left(\dfrac{\sqrt{2mE}}{\hbar}r\right) + \\ C_2 \dfrac{\sqrt{2mE}}{\hbar} y_l\left(\dfrac{\sqrt{2mE}}{\hbar}r\right)\right] & : \quad r \geq r_c \end{cases} \tag{6}$$

Where for the radius larger than cut-off radius r_c the solutions are the Coulomb functions. This area is also represented by the solution for the wave function that has exact analytical form.

And finaly for the energies larger than E_{co} the solution is

$$P(r) = \begin{cases} C_0 F_l(\eta, r) & : \quad r < r_c \\ C_0 r\left[C_1 \dfrac{\sqrt{2mE}}{\hbar} j_l\left(\dfrac{\sqrt{2mE}}{\hbar}r\right) + \\ C_2 \dfrac{\sqrt{2mE}}{\hbar} y_l\left(\dfrac{\sqrt{2mE}}{\hbar}r\right)\right] & : \quad r \geq r_c \end{cases} \tag{7}$$

Here $\eta = 1/\sqrt{2(E - 1/r_c)}$.

The presented results emphasize the usability of this model. It has been tested and proofed that all of the calculated cross sections converge towards its theoretical values from [1]. Also the form of the potential (1) gives the unique opportunity to transfer from the dipole matrix element calculation to the gradient of the potential for the calculation of a cross sections. Because of that the integration for the free-free cross section calculations are not approximated in any way.

The influence of collective phenomena in plasma is introduced here with the help of broadening of bound level states, and additional shifting of bound states. Both of the effects are considered to be external parameters in this model. The broadening is modeled without a further physical meaning as a Lorenz form broadening.

The real behavior of the calculated data in comparison to the real ones from [2] are represented in Fig. 1.

Absorption coeficient $N_e = 1.5\ 10^{19}$ cm^{-3} T = 23000 K r_c = 55 a.u.
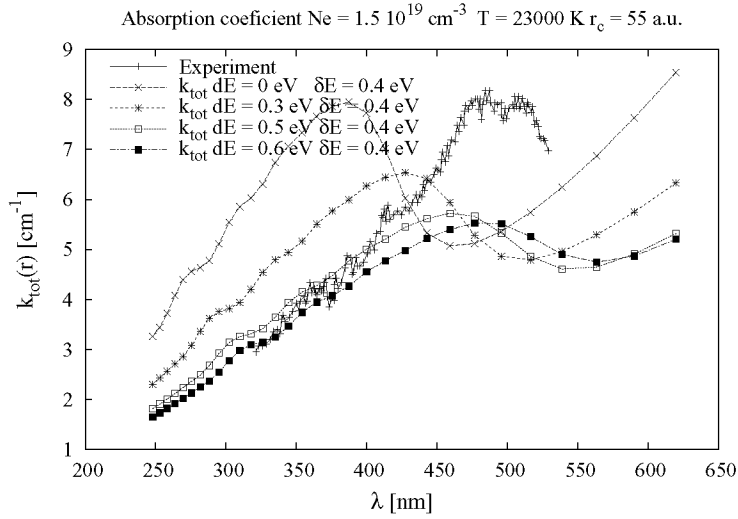
FIGURE 1. The comparison of calculated data with the real experimental data, The parameters of the curves are given with the δE for the Lorenz FWHM and dE for the additional shift of the levels.

The modification of a Coulomb cut-off potential

The data in Fig. 1 revealed the fact that the calculations for the broadening of 0.4 eV and without any shift has the form very similar to the real experimental curve. The only problem is that for the lower part of the wavelengths it is expected to have continuous spectra of photo-absorption as a dominant process, i.e. it is not expected to have any line from bound-bound transition in that range.

The need for more precise potential of inter-plasma interaction occurred. One of the most promising potentials for the description is the additional shifted Coulomb cut-off potential given by the equation

$$U(r) = \begin{cases} -\dfrac{e^2}{r} + k_{rc}\dfrac{e^2}{r_c} & : & 0 < r \le r_c \\ 0 & : & r_c < r \end{cases} \tag{8}$$

With the help of such a potential the form of the solutions is consistent, i.e. all of the mathematical procedures used for the calculation of the cross section with the help of the potential (1) are easily adopted for the new potential (8). The form of the potential (8) enables us to perform an exact calculation for the free-free cross section, as it was mentioned for the previous potential. So without any loss of the advantages of the first potential (1) the new one has included additional shifting of the levels (dE in Fig. 1) inside of the potential itself with the help of a parameter k_{rc}. The value of the k_{rc} should be varied in the area close to the theoretical value $k_{rc} = 5/2$. The areas of the solution

are given by

a) $E < 0$
b) $0 < E < k_{rc}e^2/r_c$
c) $k_{rc}e^2/r_c < E$

and the solutions are listed below, for the bound states

$$P(r) = \begin{cases} C_N M_{\kappa,\mu}\left(\dfrac{2r}{a_0 v^2}\right) & : \quad r < r_c \\ \sqrt{r} C_N C_{E,l} k_{l+1/2}\left(r\sqrt{-2mE/\hbar^2}\right) & : \quad r \geq r_c \end{cases} \tag{9}$$

Here $\mu = l + 1/2$, $v = \sqrt{-\dfrac{e^2}{a_0}\dfrac{1}{2E_c}}$, $a_0 = \hbar^2/me^2$ and $E_c = E + k_{rc}e^2/r_c < 0$. We note that the difference in the form of a potential reflects only on definition of a corrected energy E_c.

The form of solutions also does not differ for the first area of the solutions for the free states $(0 \leq E < E_{co})$

$$P(r) = \begin{cases} C_0 M_{\kappa,\mu}\left(\dfrac{2r}{a_0 v^2}\right) & : \quad r < r_c \\ C_0 r\left[C_1 \dfrac{\sqrt{2mE}}{\hbar} j_l\left(\dfrac{\sqrt{2mE}}{\hbar}r\right) + \\ C_2 \dfrac{\sqrt{2mE}}{\hbar} y_l\left(\dfrac{\sqrt{2mE}}{\hbar}r\right)\right] & : \quad r \geq r_c \end{cases}. \tag{10}$$

The same could be sad for the upper part of the free states energy $E > E_{co}$

$$P(r) = \begin{cases} C_0 F_l(\eta, r) & : \quad r < r_c \\ C_0 r\left[C_1 \dfrac{\sqrt{2mE}}{\hbar} j_l\left(\dfrac{\sqrt{2mE}}{\hbar}r\right) + \\ C_2 \dfrac{\sqrt{2mE}}{\hbar} y_l\left(\dfrac{\sqrt{2mE}}{\hbar}r\right)\right] & : \quad r \geq r_c \end{cases}, \tag{11}$$

where $\eta = 1/\sqrt{2(E - k_{rc}1/r_c)}$.

The value of the shift

With the analysis of the data presented in Fig. 1 it could be seen that the theoretical value for no additional shift has similar shape to the experimental one and is displaced only for approximately 0.75 eV. Up to now the potential (8) is not completely introduced in the calculation. The work is in progress. The calculation of a bound state energies and wave functions are implemented. The comparison of the values for the energy levels for the potential (1) - column E_1 and for (8) - column E_2 are presented in Table 1
It could be easily seen that for the theoretical value $k_{rc} = 5/2$ the difference between the levels values is close to 0.75 eV.

TABLE 1. Comparison of the energy levels for the potentials (1) and (8).

n	l	$E_1 \ [eV]$	$E_2 \ [eV]$
1	0	-13.1107	-12.3687
2	0	-2.90672	-2.16467
2	1	-2.90645	-2.16467
3	0	-1.01716	-0.275107
3	1	-1.01716	-0.275107
3	2	-1.01683	-0.275047

CONCLUSIONS

The presented method is in a good agreement with the experimental data. There is a possibility that the use of the modified potential (8) should give even better results. The usage of the Coulomb cut-off potential, see [3–7] for example, has led to a good agreement for the transport coefficients of plasma. However the optical properties of those dense plasma are not covered in such manner. This model could led to a unified description of plasma optical and transport properties.

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